

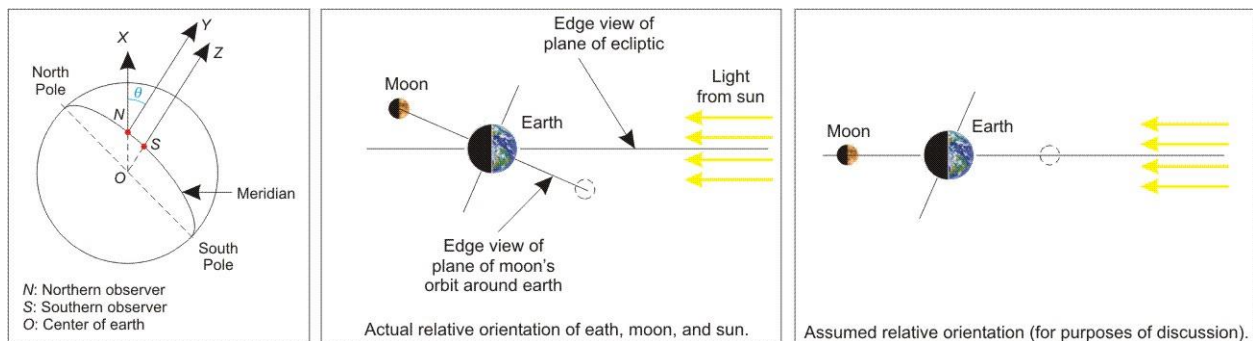
## Simple astronomy within the solar system

Adapted 2013 by Ruediger Kuhnke from:

Kittel, C; Knight, W D; Ruderman, M A.: Mechanics. Berkeley Physics Course Volume 1. New York 1970

Two amateur astronomers set for themselves the goal of determining the diameter and mass of the sun. After giving the problem considerable thought, they realize that, to begin with, several subsidiary quantities must be established.

Their first step is to determine the radius of the earth, which they accomplish by using accurate maps to locate themselves 500 miles apart on a geographical meridian (line of longitude). They communicate by cellphone. The southern observer S selects a star which passes through his zenith at some instant of time.

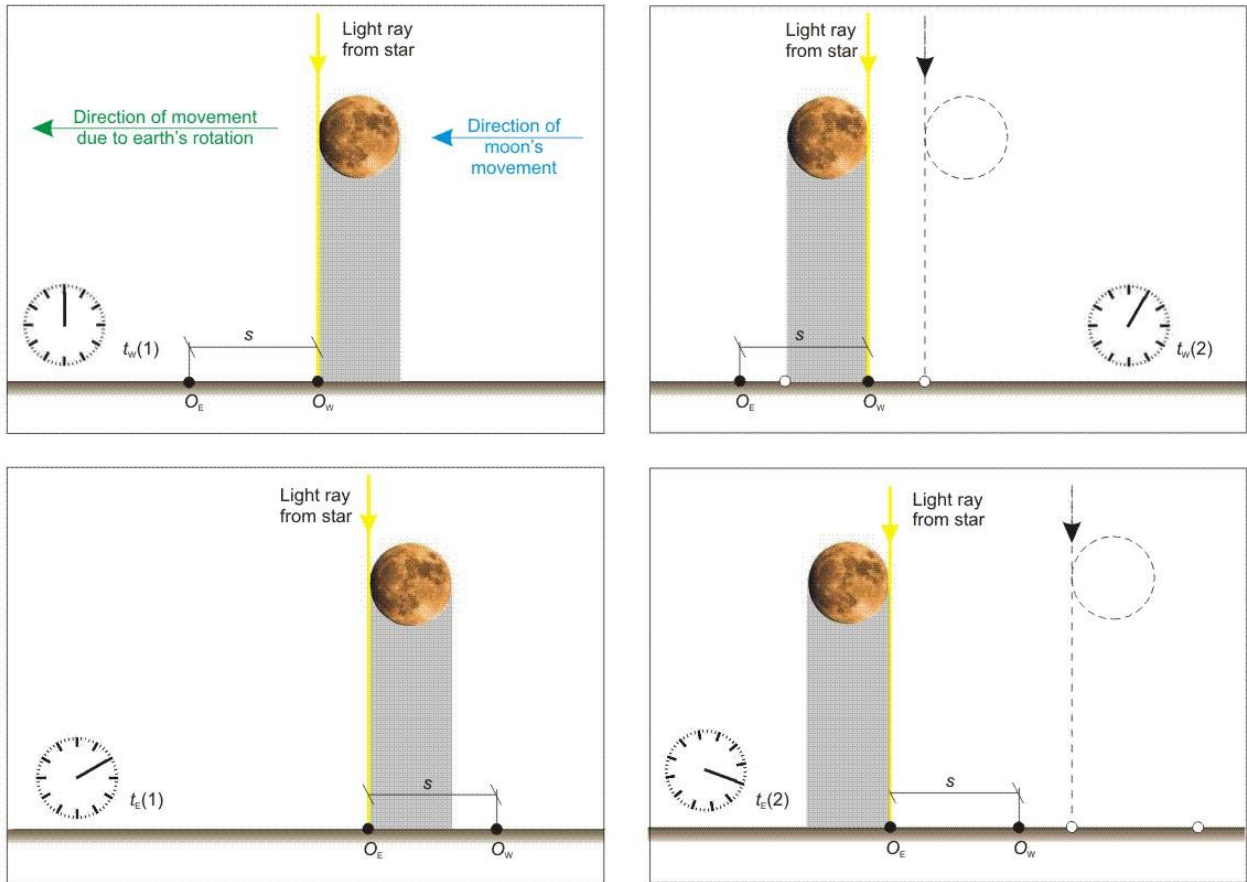


Light from distant star at zenith is received by S along line ZSO (line from zenith for S). Light from same star is received by N along line YN  $\parallel$  ZSO. The line XNO is directed from the zenith for N. YN and XNO are inclined at zenith angle  $\theta$ .

At the time the selected star is passing through the zenith of the southern observer, it is also crossing the meridian of the northern observer N, but as a consequence of the earth's curvature, it crosses below his zenith.

(a) If the northern observer notes a zenith angle of  $\theta = 7.2^\circ$ , show that the radius of the earth is calculated to be  $6.4 \cdot 10^8$  cm.

The two amateurs find that the second step necessary is to measure the velocity of the moon in its orbit about the center of the earth. They accomplish this indirectly by determining the times at which some particular star, as seen from two different points on the earth's surface, is eclipsed by the moon. To reduce the problems they make a number of simplifying assumptions: the moon and the star are located in the ecliptic (the plane of the sun and of the earth's orbit); the star passes directly behind the moon, i. e., along a diameter; the observations are made at midnight with a full moon. The curvature of the earth, effects of atmospheric refraction, and further corrections are neglected. The geometrical situation is pictured above. Parallel rays of light from a distant star are received by the two observers  $O_W$  and  $O_E$ . The westerly observer  $O_W$  notes the time  $t_W(1)$  at which the star disappears behind the moon and later the time  $t_W(2)$  at which it reappears. Similar observations  $t_E(1)$  and  $t_E(2)$  are made by the easterly observer.



(1) Position of moon and Observers  $O_w$  and  $O_E$  at time  $t_w(1)$ . Starlight is assumed to come from an infinite distance and therefore has parallel rays. (2) At time  $t_w(2)$  the moon has moved to this position, and starlight now appears to Observer  $O_w$  who, because of the earth's rotation, has moved in the meantime to his new position shown here. (3) In the same way,  $O_E$  sees star disappear at  $t_e(1)$  and after an interval... (4) reappear to him at time  $t_e(2)$ .

(b) Show that the velocity of the moon relative to the center of the earth is given by the expression

$$v_M = v_0 + \frac{S}{t_E(1) - t_W(1)}$$

where  $S$  is the separation of the observers,  $v_0$  the linear velocity of the observers,  $v_M$  the linear velocity of the moon, and  $t_E(1)$  and  $t_W(1)$  are as defined above.

Knowing the velocity of the moon, either observer will be able to determine the moon's diameter from the total time that the moon eclipses the star.

(c) Show that the moon's diameter is

$$2R_M = (v_M - v_0)(t_2 - t_1)$$

where  $t_1$  and  $t_2$  are the set of times measured by either observer.

(d) From the radius of the earth determined above and the earth's known period of rotation determine its surface velocity at  $30^\circ$  N latitude relative to its center.

Ans.  $4.04 \cdot 10^4$  cm/sec

Typical observations which might have been obtained by the two amateurs are given in the following table.

Observation	$t_w(1)$	$t_w(2)$	$t_e(1)$	$t_e(2)$
Time, min	0,0	95.6	22.0	117.7

(e) Use these data and previous results to show that the velocity of the moon is  $10.1 \cdot 10^4$  cm/sec and its diameter is  $3.48 \cdot 10^8$  cm. Having determined the moon's orbital velocity, the two amateurs quickly use its known orbital period ( $2.36 \cdot 10^6$  sec) to determine the radius of its orbit.

(f) Show that their observations lead to a result in agreement with accurate measurements ( $3.8 \cdot 10^{10}$  cm).

If the orbit of a satellite is circular, it is quite simple to determine the mass of the attracting body. Using Newton's law of gravitation  $F = GM_E M_M / r^2$  for the force between the earth and the moon one may show that

$$GM_E = v_M^2 r = R^2 g$$

where  $G$  is the gravitational constant,  $M_E$  the mass of the earth,  $v_M$  the velocity of the moon,  $r$  the radius of the moon's orbit,  $R$  the radius of the earth, and  $g$  the acceleration of gravity at the earth's surface ( $980$  cm/sec<sup>2</sup>). The first equality above is obtained by equating the gravitational force to the centrifugal force  $M_M v_M^2 / r$ , where  $M_M$  is the mass of the moon.

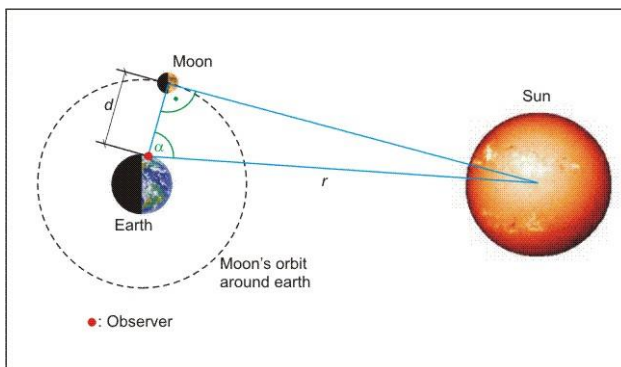
(g) Calculate the magnitude of the constant  $GM_E$ .

After considerable effort the amateurs conclude that, because they do not have sufficient information to calculate the earth's mass, they must perform another experiment. Ideally, they should measure the gravitational constant  $G$ . Because this is a fairly difficult experiment to carry out, they decide instead to estimate the density of the earth. From a study of surface materials they obtain a value of  $5$  gm/cm<sup>3</sup>.

(h) What is the approximate mass of the earth on this basis? To what percentage error does this actually correspond?

(i) Use this estimate and preceding results to obtain the order of magnitude of  $G$ .

Next the two amateurs determine the distance to the sun by using the known data about the moon's orbit as shown in the illustration. At a particular time the sun, moon, and earth are in positions such that the shadow edge coincides with a major diameter of the moon (which to an observer appears to be exactly a half moon). At this time the observer determines the angle  $\alpha$  between lines to the moon and to the sun.



Method of determining distance from earth to sun using known data about the moon's orbit

(j) For an angle  $\alpha = 89^\circ 51'$ , calculate the distance  $r$  from the earth to the sun. Knowing the distance of the earth from the sun, the two amateurs realize that Kepler's third law (in addition to Newton's law of motion) will allow them to calculate the mass of the sun.

(k) Calculate the mass of the sun.

Finally the two amateurs, who had been fortunate enough to view the 1963 total eclipse of the sun, remember that the moon almost exactly obscured the sun. Calculate the radius  $R_S$  of the sun.

(l) Determine the diameter of the sun on the basis of information obtained above.